Problem 1. In a binary min-heap with $n$ elements, both the Insert and Extract-Min operations take $O(\log n)$ worst-case time. Give a potential function $\Phi$ and prove that using $\Phi$, the amortized cost of Insert is $O(\log n)$ and the amortized cost of Extract-Min is $O(1)$.

Problem 2. Suppose that instead of contracting a dynamic table by halving its size when its load factor drops below 1/4, we contract it by multiplying its size by 2/3 when its load drops below 1/3. Using the potential function

$$\Phi(T) = |2 \cdot T.num - T.size|,$$

show that the amortized cost of a Table-Delete that uses this strategy is $O(1)$.

Problem 3. For any integer $n > 1$, give a sequence of operations performed on an empty Fibonacci heap $H$, such that the resulting heap contains a single tree that is a linear chain of $n$ nodes (that is, a tree with $n$ nodes, and of height $n - 1$).