6331 - Algorithms, Spring 2014, CSE, OSU
Homework 5
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Problem 1. In a binary min-heap with n elements, both the INSERT and EXTRACT-MIN operations take $O(\log n)$ worst-case time. Give a potential function Φ and prove that using Φ, the amortized cost of INSERT is $O(\log n)$ and the amortized cost of EXTRACT-MIN is $O(1)$.

Problem 2. Suppose that instead of contracting a dynamic table by halving its size when its load factor drops below $1/4$, we contract it by multiplying its size by $2/3$ when its load drops below $1/3$. Using the potential function

$$\Phi(T) = |2 \cdot T.num - T.size|,$$

show that the amortized cost of a TABLE-DELETE that uses this strategy is $O(1)$.

Problem 3. For any integer $n > 1$, give a sequence of operations performed on an empty Fibonacci heap H, such that the resulting heap contains a single tree that is a linear chain of n nodes (that is, a tree with n nodes, and of height $n - 1$).