6331 - Algorithms, Spring 2014, CSE, OSU
Homework 7
Instructor: Anastasios Sidiropoulos

Problem 1. Let G be an undirected graph, with weights on the edges.

(a) Let $s \in V(G)$, and let T be a shortest-paths tree with root s. Is it true that T must also be a minimum spanning tree? Give a proof that justifies your answer.

(b) Let $s \in V(G)$, and let T be a minimum spanning tree. Is it true that T must also be a shortest-paths tree with root the given vertex s? Give a proof that justifies your answer.

Problem 2. Show that if all edge weights of a graph are non-negative, then the weight of any minimum spanning tree is equal to the minimum total weight of any subset of edges that connects all vertices. Does this remain true when the weights can be negative?

Problem 3. Given a graph G and a minimum spanning tree T of G, suppose that we decrease the weight of one of the edges $e = \{u, v\}$ not in T. Let G' be the resulting modified graph. Let P be the unique path between u and v in T. Let C be cycle formed by the union of P with e. Let e' be an edge in C of maximum weight. Let $T' = (T \cup \{e\}) \setminus \{e'\}$. Prove, or disprove that T' is a minimum spanning tree of G'.