

6332 - Advanced algorithms, Spring 2015, CSE, OSU

Homework 4

Instructor: Anastasios Sidiropoulos

Due date: April 22, 2015

Problem 1. (Set Basis) The parameterized Set Basis problem is defined as follows:

Input: A collection C of subsets of a finite set S .
Parameter: An integer $k > 0$.
Question: Is there a collection B of subsets of S with $|B| = k$ such that for every $A \in C$, there exists a sub collection of B whose union is exactly A ?

Show that Set Basis is fixed parameter tractable (FPT).

Problem 2. (Losing your marbles) The parameterized Weighted Marbles problem is defined as follows:

Input: A sequence of marbles, each with an integer weight and a color.
Parameter: An integer $k > 0$.
Question: Can we remove marbles of total weight at most k , such that for each color, all marbles of that color are consecutive?

Show that Weighted Marbles is FPT.

Optional problem 3. (TSP on bounded treewidth graphs) Recall that an input to the Traveling Salesperson Problem (TSP) consists of a connected undirected n -vertex graph $G = (V, E)$ together with a non-negative function $w : E \rightarrow \mathbb{R}_{\geq 0}$, where for any $\{u, v\} \in E$, $w(\{u, v\})$ is the *length* of the edge $\{u, v\}$. The goal is to find a *tour* of minimum length that visits all vertices and returns to the starting vertex.

Show that TSP is fixed parameter tractable when the parameter is the treewidth of G . That is, show that there exists an algorithm for TSP on graphs of treewidth k with running time $f(k) \cdot n^{O(1)}$ for some function f .