6331 - Algorithms, Autumn 2016, CSE, OSU
Homework 7
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Problem 1. In a binary min-heap with n elements, both the INSERT and EXTRACT-MIN operations take $O(\log n)$ worst-case time. Give a potential function Φ and prove that using Φ, the amortized cost of INSERT is $O(\log n)$ and the amortized cost of EXTRACT-MIN is $O(1)$.

Problem 2. For any integer $n > 1$, give a sequence of operations performed on an empty Fibonacci heap H, such that the resulting heap contains a single tree that is a linear chain of n nodes (that is, a tree with n nodes, and of height $n - 1$).

Problem 3. We are interested in designing a data structure for maintaining a set A of integers, that supports the following operations:

- Insert(A, x): Insert the integer x into the set A.
- ApproximateMedian(A): Return some $x \in A$ such that at least 25% of the elements in A are not greater than x and at least 25% of the elements in A are not smaller than x.

You may assume that the data structure starts with the set A being empty.

(a) Design a data structure for the above problem using a balanced binary search tree, with worst-case query time $O(\log n)$.

(b) Design a data structure for the above problem using an array, with amortized time per query $O(\log n)$.