Problemset 3
TTIC 31100 / CMSC 39000 Computational geometry

November 8, 2010

Problem 1. Recall that ℓ_1^d denotes the space $\mathbb{R}^d$, endowed with the ℓ_1 norm. Describe an algorithm which given a set of n points in ℓ_1^d, computes their diameter in time $O(2^d \cdot n)$. Hint: Use an embedding into ℓ_∞ of appropriate dimension.

Problem 2. Show that any embedding of the n-cycle into the line, has distortion $\Omega(n)$.

Problem 3. Recall that $K_{3,3}$ is the complete bipartite graph with each side having 3 vertices. Let G be the graph obtained from $K_{3,3}$ after replacing every edge with a path of length n. Show that any embedding of the shortest-path metric of G into the Euclidean plane, has distortion $\Omega(n)$.

Problem 4. Let (X, d) be the uniform metric on n points. I.e. $X = \{x_1, \ldots, x_n\}$, and for any $i \neq j \in \{1, \ldots, n\}$, $d(x_i, x_j) = 1$. Show that for any fixed $d \geq 1$, any embedding of (X, d) into $\mathbb{R}^d$, has distortion $\Omega(n^{1/d})$. Hint: It might be helpful to prove the assertion first for the case $d = 1$.